Unit 3:
L-2
Statistical Quality Control

Learning Objectives

A **control chart** is a graphical tool for monitoring the activity of an ongoing process. Control charts are sometimes referred to as **Shewhart control charts,** because Walter A. Shewhart first proposed their general theory. The values of the quality characteristic are plotted along the vertical axis, and the horizontal axis represents the samples, or subgroups (in order of time), from which the quality characteristic is found.

More Basic Principles

- Control charts may be used to estimate process parameters, which are used to determine **capability**
- Two general types of control charts
  - Variables (lecture 6)
    - Continuous scale of measurement
    - Quality characteristic described by central tendency and a measure of variability
  - Attributes (lecture 7)
    - Conforming/nonconforming
    - Counts
- **Control chart design** encompasses selection of sample size, control limits, and sampling frequency

Types of Process Variability

- **Stationary** and **uncorrelated** – data vary around a fixed mean in a stable or predictable manner
- **Stationary** and **autocorrelated** – successive observations are dependent with tendency to move in long runs on either side of mean
- **Nonstationary** – process drifts without any sense of a stable or fixed mean

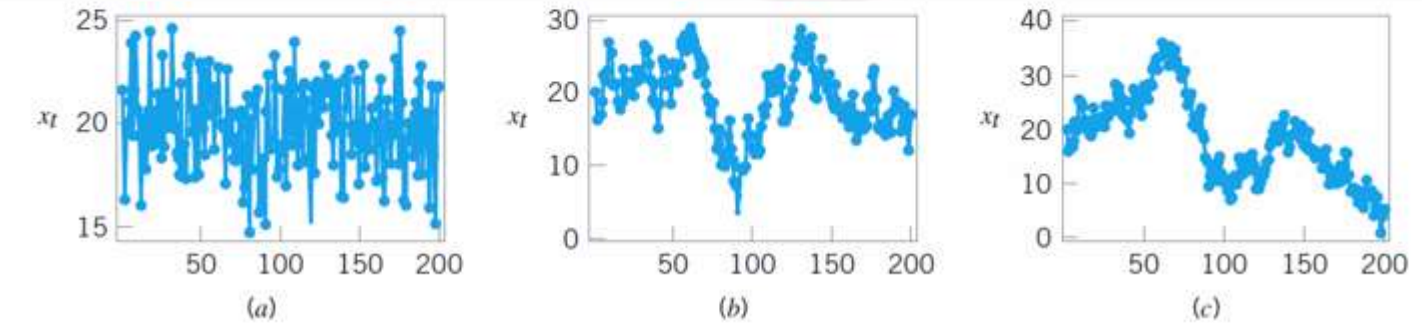

Fig: Data of different processes

Reasons for Popularity of Control Charts

1. Control charts are a proven technique for improving productivity.
2. Control charts are effective in defect prevention.
3. Control charts prevent unnecessary process adjustment.
4. Control charts provide diagnostic information.
5. Control charts provide information about process capability.

Choice of control limit

- 3-Sigma Control Limits
  - Probability of type I error is 0.0027
- Probability Limits
  - Type I error probability is chosen directly
  - For example, 0.001 gives 3.09-sigma control limits
- Warning Limits
  - Typically selected as 2-sigma limits

Sample Size and Sampling Frequency

Another way to evaluate the decisions regarding sample size and sampling frequency is through the **average run length (ARL)** of the control chart. Essentially, the ARL is the average number of points that must be plotted before a point indicates an out-of-control condition. If the process observations are uncorrelated, then for any Shewhart control chart, the ARL can be calculated easily from

$$\boxed{\text{ARL} = \frac{1}{p} \qquad (5.2)}$$

where p is the probability that any point exceeds the control limits. This equation can be used to evaluate the performance of the control chart.

To illustrate, for the $\bar{x}$ chart with three-sigma limits, $p = 0.0027$ is the probability that a single point falls outside the limits when the process is in control. Therefore, the average run length of the $\bar{x}$ chart when the process is in control (called ARL_0) is

$$\mathrm{ARL}_0 = \frac{1}{p} = \frac{1}{0.0027} = 370$$

That is, even if the process remains in control, an out-of-control signal will be generated every 370 samples, on the average.

The use of average run lengths to describe the performance of control charts has been subjected to criticism in recent years. The reasons for this arise because the distribution of run length for a Shewhart control chart is a geometric distribution (refer to Section 2-2.4). Consequently, there are two concerns with ARL: (1) the standard deviation of the run length is very large, and (2) the geometric distribution is very skewed, so the mean of the distribution (the ARL) is not necessarily a very "typical" value of the run length.

For example, consider the Shewhart $\bar{x}$ control chart with three-sigma limits. When the process is in control, we have noted that $p = 0.0027$ and the in-control ARL_0 is $ARL_0 = 1/p = 1/0.0027 = 370$. This is the mean of the geometric distribution. Now the standard deviation of the geometric distribution is

$$\sqrt{(1-p)}\Big/p = \sqrt{(1-0.0027)}\Big/0.0027 \cong 370$$

That is, the standard deviation of the geometric distribution in this case is approximately equal to its mean. As a result, the actual ARL_0 observed in practice for the Shewhart $\bar{x}$ control chart will likely vary considerably. Furthermore, for the geometric distribution with $p = 0.0027$, the 10th and 50th percentiles of the distribution are 38 and 256, respectively. This means that approximately 10% of the time the in-control run length will be less than or equal to 38 samples and 50% of the time it will be less than or equal to 256 samples. This occurs because the geometric distribution with $p = 0.0027$ is quite skewed to the right.

Consider the hard-bake process discussed earlier, and suppose we are sampling every hour. Equation (5.3) indicates that we will have a **false alarm** about every 370 hours on the average.

Now consider how the control chart performs in detecting shifts in the mean. Suppose we are using a sample size of $n = 5$ and that when the process goes out of control the mean shifts to 1.725 microns. From the operating characteristic curve in Fig. 5.9 we find that if the process mean is 1.725 microns, the probability of $\overline{x}$ falling between the control limits is approximately 0.35. Therefore, p in equation (5.2) is 0.35, and the out-of-control ARL (called ARL_1) is

$$\text{ARL}_1 = \frac{1}{p} = \frac{1}{0.35} = 2.86$$

That is, the control chart will require 2.86 samples to detect the process shift, on the average, and since the time interval between samples is $h = 1$ hour, the average time required to detect this shift is

$$\text{ATS} = \text{ARL}_1 h = 2.86\,(1) = 2.86 \text{ hours}$$

Suppose that this is unacceptable, because production of wafers with mean flow width of 1.725 microns results in excessive scrap costs and can result in further upstream manufacturing problems. How can we reduce the time needed to detect the out-of-control condition? One method is to sample more frequently. For example, if we sample every half hour, then the average time to signal for this scheme is ATS = $\mathrm{ARL}_1\, h = 2.86(\frac{1}{2}) = 1.43$; that is, only 1.43 hours will elapse (on the average) between the shift and its detection. The second possibility is to increase the sample size. For example, if we use $n = 10$, then Fig. 5.9 shows that the probability of $\bar{x}$ falling between the control limits when the process mean is 1.725 microns is approximately 0.1, so that $p = 0.9$, and from equation (5.2) the out-of-control ARL or ARL_1 is

$$\mathrm{ARL}_1 = \frac{1}{p} = \frac{1}{0.9} = 1.11$$

and, if we sample every hour, the average time to signal is

$$\mathrm{ATS} = \mathrm{ARL}_1 h = 1.11(1) = 1.11 \text{ hours}$$

Thus, the larger sample size would allow the shift to be detected more quickly than with the smaller one.

Thus, the larger sample size would allow the shift to be detected about twice as quickly as the old one. If it became important to detect the shift in the (approximately) first hour after it occurred, two control chart designs would work:

Design 1	**Design 2**
Sample Size: $n = 5$	Sample Size: $n = 10$
Sampling Frequency: every half hour	Sampling Frequency: every hour

Summary:

This lecture has introduced the basic concepts of control charts for statistical process control. The benefits that can be derived from using control charts have been discussed. This lecture covers the statistical background for the use of control charts, the selection of the control limits, and the manner in which inferences can be drawn from the charts. The two types of errors that can be encountered in making inferences from control charts are discussed.

References:

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