School of Mechanical Engineering

Course Code : BAUT3002 Course Name: Heat Engineering

Lecture 3

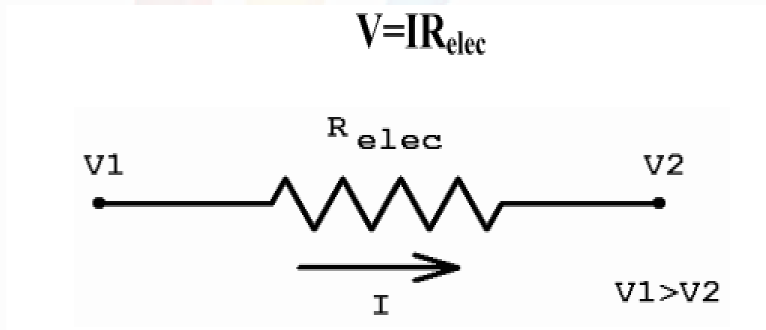
Heat engineering

Name of the Faculty: Mr. Faisal Shameem Program Name: B.Tech(ME)

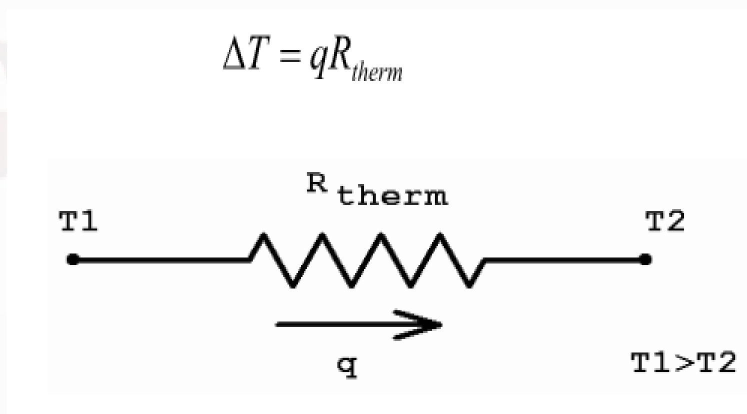
Analogy between heat and electricity flow

Thermal resistance (electrical analogy): Physical systems are said to be analogous if that obey the same mathematical equation.

Ohm’s law:



Using this terminology it is common to speak of a thermal resistance



Summary of Electrical Analogy

System	Current	Resistance	Potential Difference
Electrical	I	R	ΔV
Cartesian Conduction	q	$\frac{L}{kA}$	ΔT
Cylindrical Conduction	q	$\frac{\ln{r_2}/{r_1}}{2\pi kL}$	ΔT
Conduction through sphere	q	$\frac{1/r_1-1/r_2}{4\pi k}$	ΔT
Convection	q	$\frac{1}{h\cdot A_s}$	ΔT

Steady State One Dimensional Heat Conduction

Rectangular Coordinates (Without heat generation)

$$\frac{\partial^2 T}{\partial x^2} = 0$$

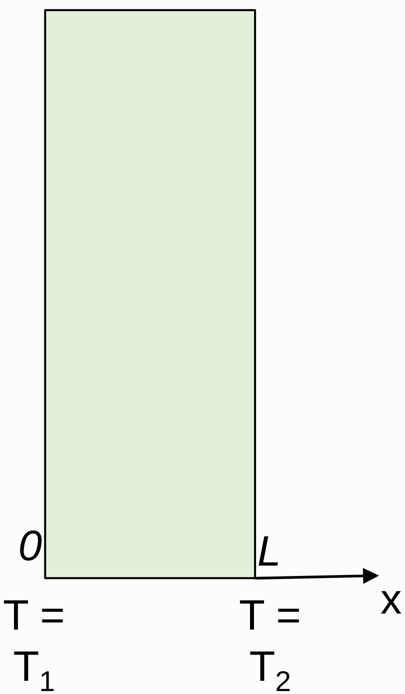
Governing Equation

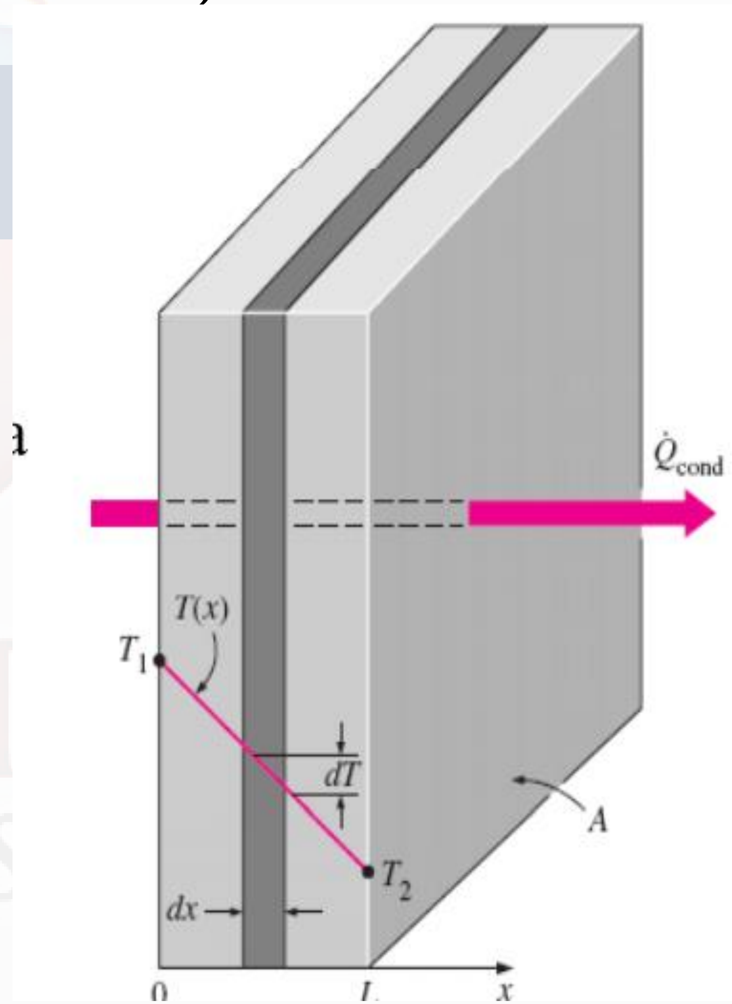
$$T(x) = c_1 x + c_2$$

$$T(x) = \left(\frac{T_2 - T_1}{L} \right) x + T_1$$

$$Q_x = \frac{K.A.(T_1 - T_2)}{L}$$

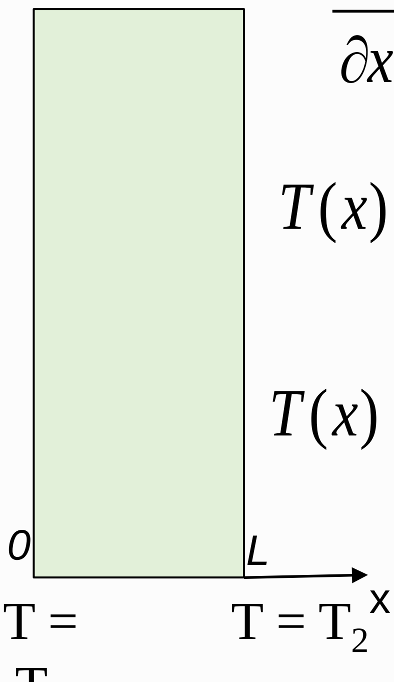
$$R = \frac{L}{K.A}$$





Steady State One Dimensional Heat Conduction

Rectangular Coordinates (With heat generation)

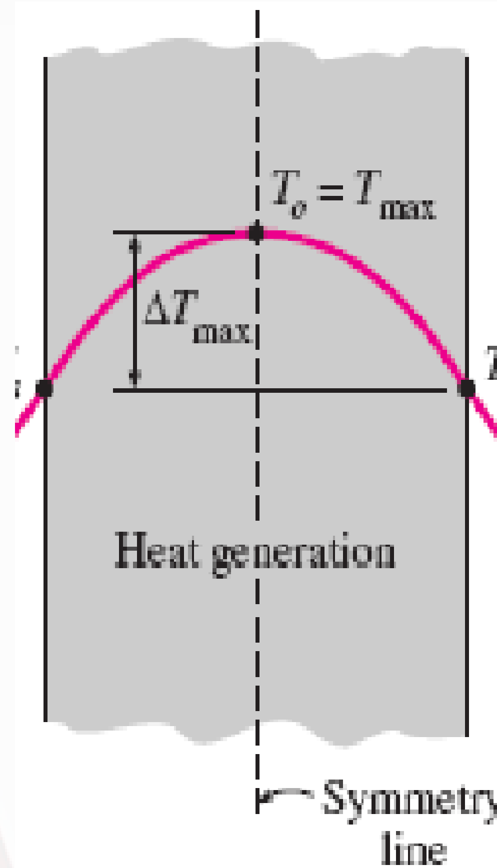


$$\frac{\partial^2 T}{\partial x^2} + \frac{g_0}{k} = 0$$

Governing Equation

$$T(x) = -\frac{g_0}{2k}x^2 + c_1 x + c_2$$

$$T(x) = -\frac{g_0}{2k}x^2 + \left(\frac{T_2 - T_1}{L} + \frac{g_0}{2k}L\right)x + T_1$$

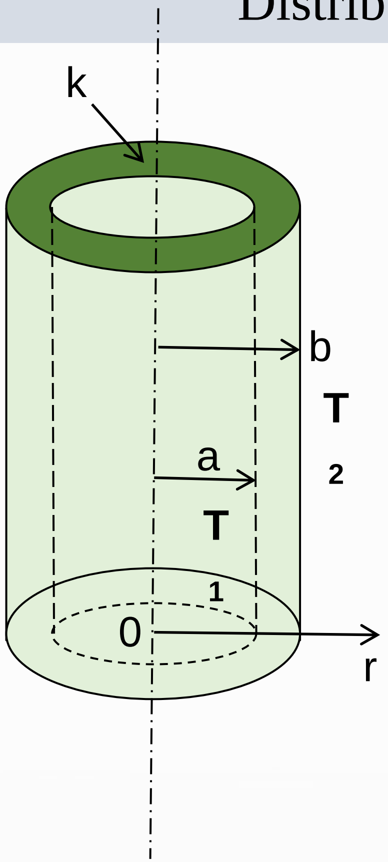


Maximum temperature is obtained at the middle of the wall and can be obtained by differentiating the above temperature distribution equation.

Steady State One Dimensional Heat Conduction

Cylindrical Coordinates (Hollow Cylinder)

Determination of Temperature Distribution



Mathematical formulation of this problem is

$$\frac{d}{dr}\left[r\frac{dT(r)}{dr}\right]=0 \quad \text{in a < r < b}$$

$$T(r)=c_1\ln r+c_2$$

Solving,

$$c_1=\frac{T_2-T_1}{\ln(b/a)}$$

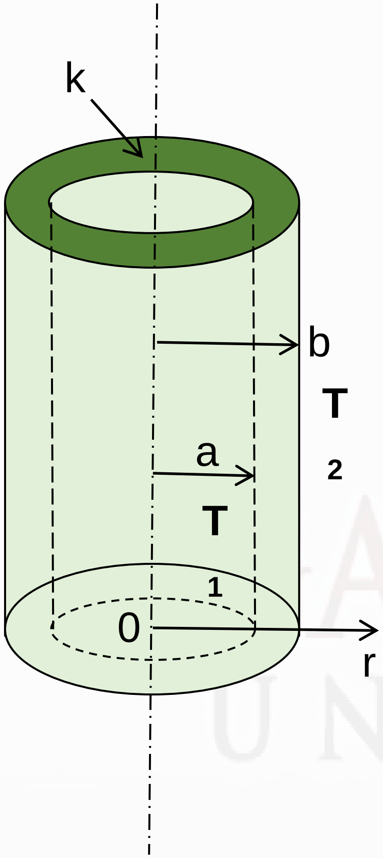
$$c_2=T_1-(T_2-T_1)\frac{\ln(a)}{\ln(b/a)}$$

$$\frac{T(r)-T_1}{T_2-T_1}=\frac{\ln(r/a)}{\ln(b/a)}$$

Steady State One Dimensional Heat Conduction

Expression for radial heat flow Q over a length L

Cylindrical Coordinates (Hollow Cylinder)



The heat flow is determined from,

$$Q = q(r).area = -k\frac{dT(r)}{dr}2\Pi rL$$

$$= -k2\Pi Lc_1$$

Since, $dT(r)/dr = (1/r)c_1$

$$Q = \frac{2\Pi kL}{\ln(b/a)}(T_1 - T_2)$$

Rearranging,

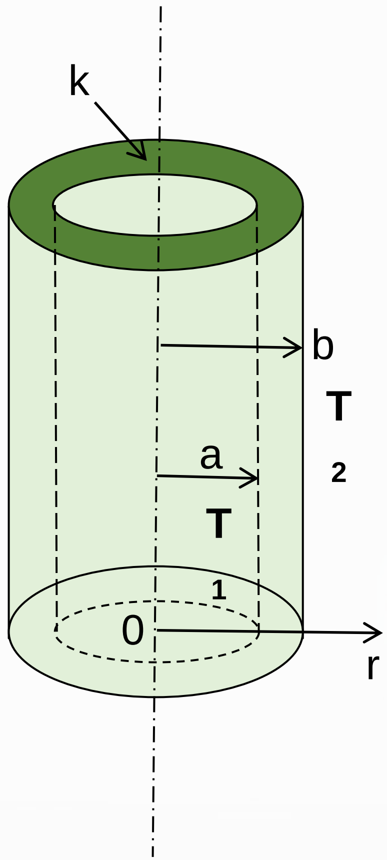
$$Q = \frac{T_1 - T_2}{R}$$

where, $R = \frac{\ln(b/a)}{2\Pi kL}$

Steady State One Dimensional Heat Conduction

Cylindrical Coordinates (Hollow Cylinder)

Expression for thermal resistance for length H



$$R = \frac{\ln(b/a)}{2\Pi kL}$$

Above equation can be rearranged as,

$$R = \frac{\ln(b/a)}{2\Pi kL} = \frac{(b-a)\ln[2\Pi bL/(2\Pi aL)]}{(b-a)2\Pi Lk}$$

$$R = \frac{t}{kA_m}$$ where, $$A_m = \frac{A_1 - A_0}{\ln(A_1 - A_0)}$$

here, $A_0 = 2\pi aH$ =area of inner surface of cylinder

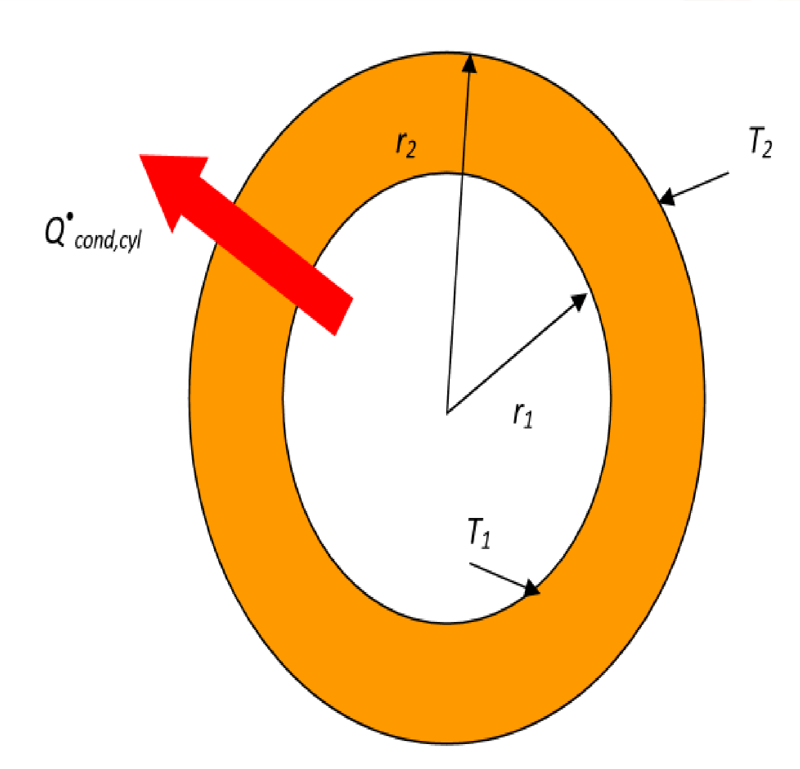
$A_1 = 2\pi bH$ =area of outer surface of cylinder

A_m = logarithmic mean area

$t = b - a$ = thickness of cylinder

Thermal resistance of hollow cylinder

Without heat generation



$$\dot{Q}_{cond,cyl} = -kA\frac{dT}{dr}$$

$$A = 2\pi rL$$

$$\int_{r1}^{r2} \frac{\dot{Q}_{cond,cyl}}{A} dr = -\int_{T1}^{T2} k\,dT \qquad A = 2\pi rL$$

$$\dot{Q}_{cond,cyl} = 2\pi kL\frac{T_1 - T_2}{\ln(r_2/r_1)}$$

$$\dot{Q}_{cond,cyl} = \frac{T_1 - T_2}{R_{cyl}}$$

$$R_{cyl} = \frac{\ln(r_2/r_1)}{2\pi kL}$$

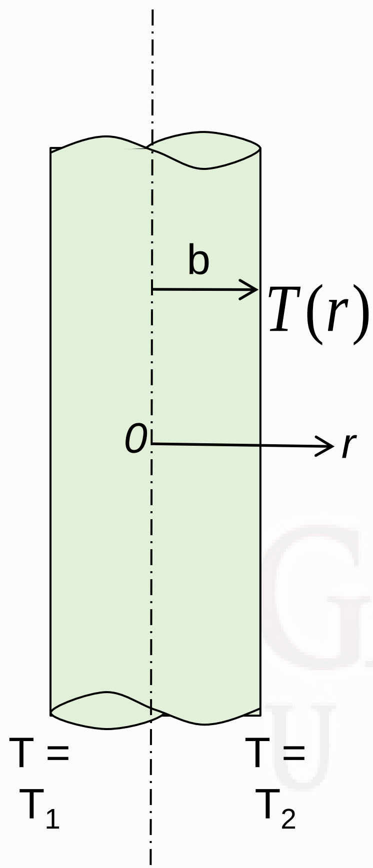
Steady State One Dimensional Heat Conduction

Cylindrical Coordinates (Solid Cylinder with heat generation

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{dT(r)}{dr}\right]+\frac{g_0}{k}=0$$

Governing Equation

$$\left[\frac{dT(r)}{dr}=0\right]_{at\ \ r=0} \quad \left[T(r)=T_2\right]_{at\ \ r=b}$$



$$T(r)=-\frac{g_0}{2k}r+c_1\ln r+c_2$$

Solving ,

$$T(r)=-\frac{g_0}{4k}\left[1-\left(\frac{r}{b}\right)^2\right]+T_2$$

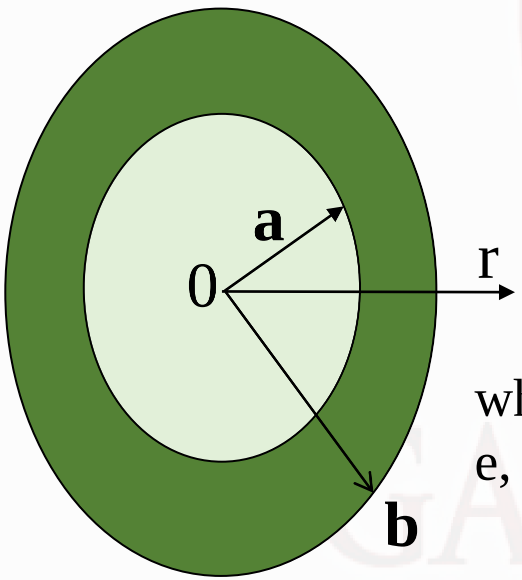
$$q(r)=-k\frac{dT(r)}{dr}=\frac{g_0 r}{2}$$

Steady State One Dimensional Heat Conduction

Spherical Coordinates (Hollow Sphere)

Expression for temperature distribution

Boundary conditions are at r=a ; T=T1 and r=b T= T2



$$\frac{d}{dr}\left(r^2\frac{dT(r)}{dr}\right)=0$$ in a < r < b

$$T(r)=-\frac{c_1}{r}+c_2$$

where,

$$c_1=-\frac{ab}{b-a}(T_1-T_2)$$

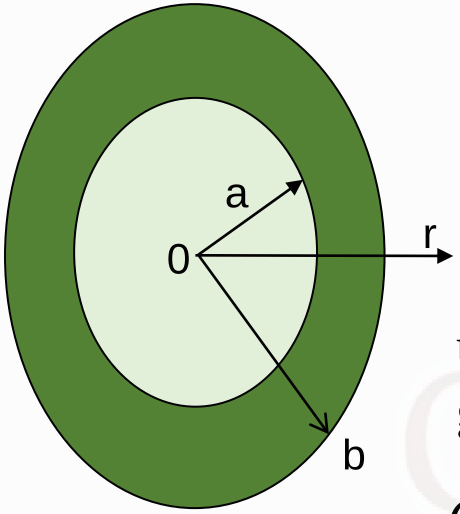
$$c_2=\frac{bT_2-aT_1}{b-a}$$

$$T(r)=\frac{a}{r}.\frac{b-r}{b-a}.T_1+\frac{b}{r}.\frac{r-a}{b-a}.T_2$$

Steady State One Dimensional Heat Conduction (Sphere)

Spherical Coordinates (Hollow Sphere)

Expression for heat flow rate Q and thermal resistance R



Heat flow rate is determined using the equation,

$$Q = (4\Pi r^2)\left[-k\frac{dT(r)}{dr}\right]$$

$$= (4\Pi r^2)\left(-k\frac{c_1}{r^2}\right) = -4\Pi k c_1$$

using, $c_1 = -\frac{ab}{b-a}(T_1 - T_2)$ from last slide

$$Q = 4\Pi k\frac{ab}{b-a}(T_1 - T_2) = \frac{T_1 - T_2}{R}$$

where, $R = \frac{b-a}{4\Pi kab}$

Thermal resistance of conduction and convection

Conduction: R = (Δx/KA)

Convection: 1/(hA)

Conductive resistance for cylinder $R = \frac{\ln(b/a)}{2\Pi kL}$

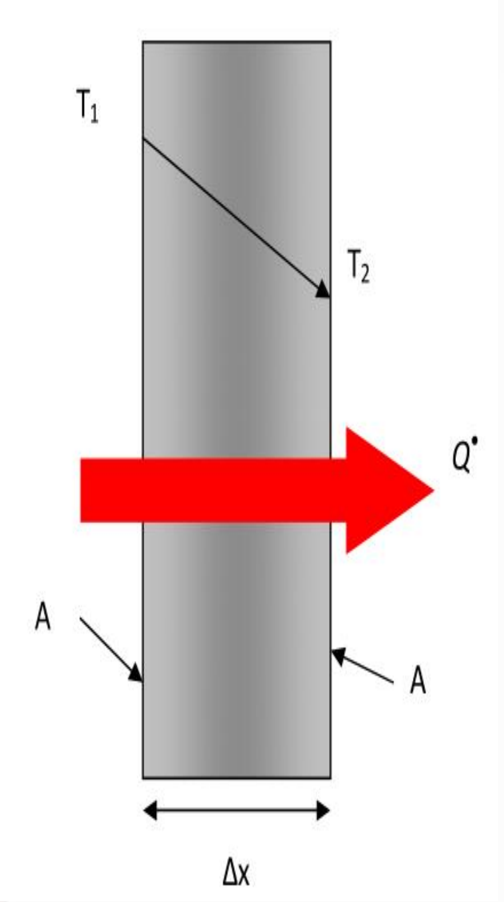
Conductive resistance for sphere $R = \frac{b-a}{4\Pi kab}$

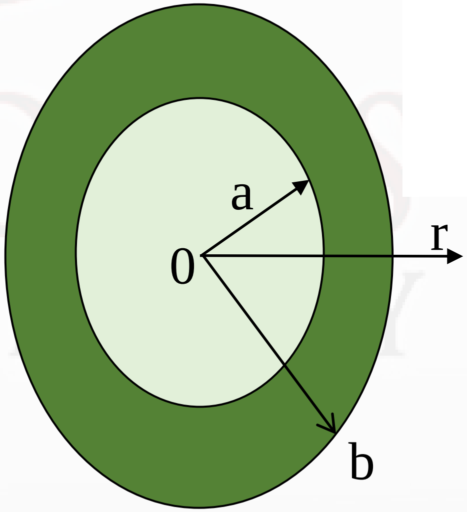
Resistance for radiation

$$\dot{Q}_{rad} = \varepsilon\sigma A\left(T_s^4 - T_\infty^4\right) = h_{rad} A\left(T_s - T_\infty\right) = \frac{T_s - T_\infty}{R_{rad}}$$

$$R_{rad} = \frac{1}{h_{rad} A}$$

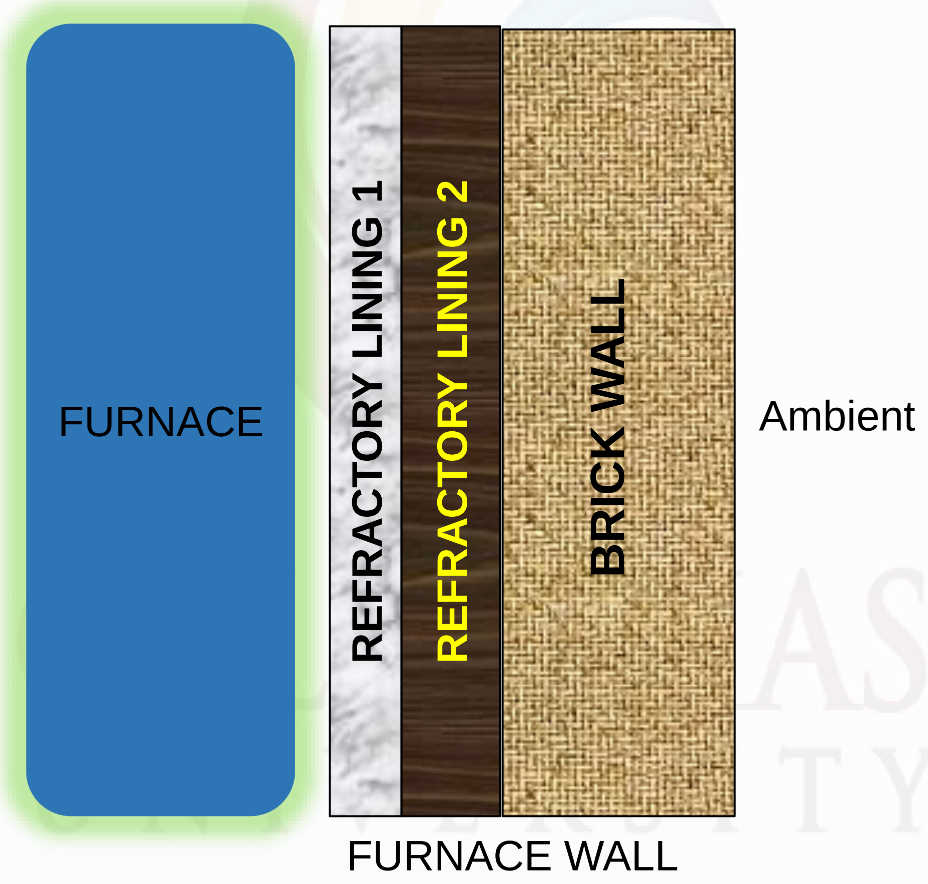
$$h_{rad} = \varepsilon\sigma\left(T_s^2 + T_\infty^2\right)\left(T_s + T_\infty\right) \qquad \left(\frac{W}{m^2K}\right)$$





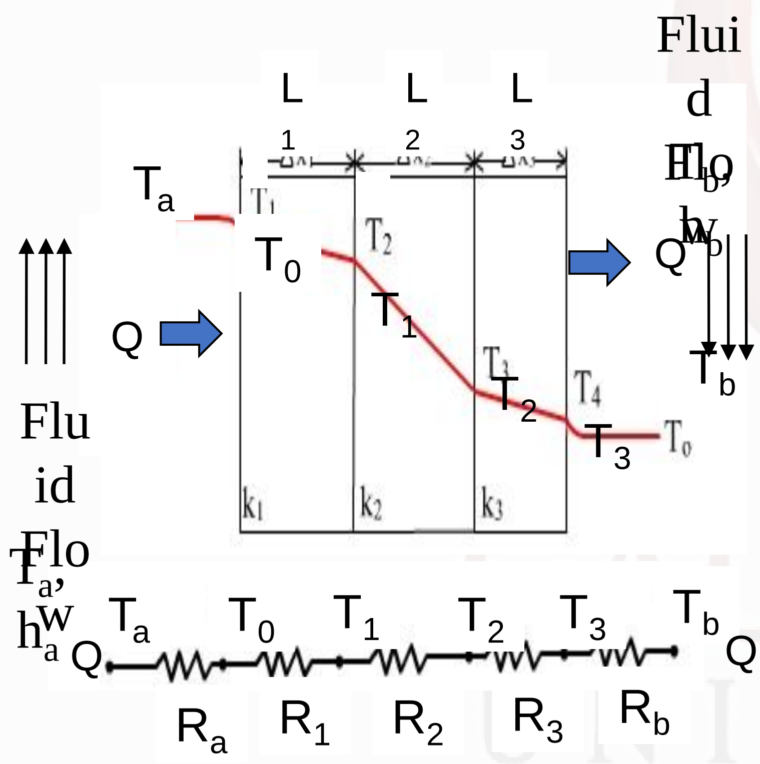
Simple and Composite Systems in rectangular

Composite system in Cartesian system



Simple and Composite Systems in rectangular

Composite system



$$Q = \frac{T_a - T_0}{R_a} = \frac{T_0 - T_1}{R_1} = \frac{T_1 - T_2}{R_2} = \frac{T_2 - T_3}{R_3} = \frac{T_3 - T_b}{R_b}$$

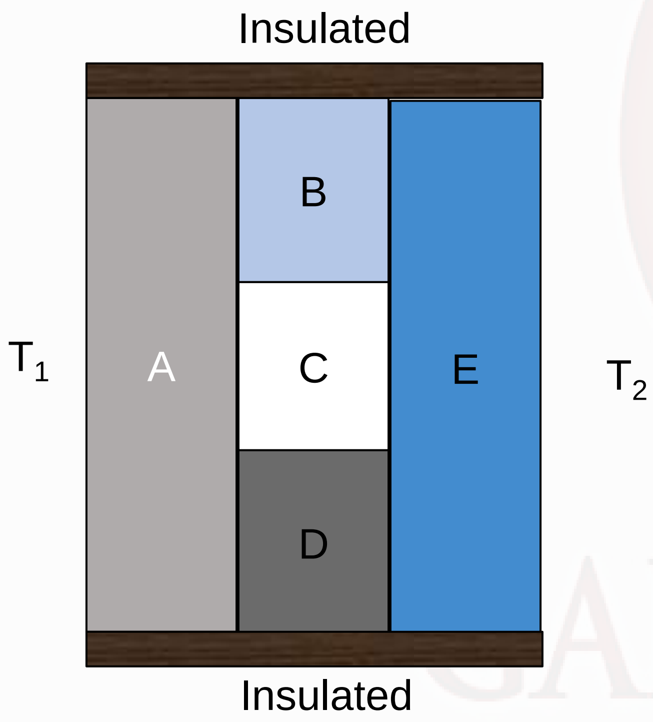
$$Q = \frac{T_a - T_b}{R} \quad W$$

$$R_a = \frac{1}{Ah_a}; R_1 = \frac{L_1}{Ak_1}; R_2 = \frac{L_2}{Ak_2}; R_3 = \frac{L_3}{Ak_3}; R_b = \frac{1}{Ah_b}$$

$$R = R_a + R_1 + R_2 + R_3 + R_b$$

Simple and Composite Systems in rectangular

Composite system (Resistance in parallel)



$$Q = \frac{T_1 - T_2}{R} \quad W$$

$$R = R_A + R_{eq.p} + R_E$$

$$\frac{1}{R_{eq.p}} = \frac{1}{R_B} + \frac{1}{R_C} + \frac{1}{R_D}$$