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Randomized quicksort

- **IDEA**: Partition around a *random* element.
- Running time is independent of the input order.
- No assumptions need to be made about the input distribution.
- No specific input elicits the worst-case behavior.
- The worst case is determined only by the output of a random-number generator.

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Randomized quicksort analysis

Let T(n) = the random variable for the running time of randomized quicksort on an input of size n, assuming random numbers are independent.

For *k* = 0, 1, ..., *n*–1, define the *indicator random variable*

 $X_{k} = \begin{cases} 1 & \text{if PARTITION generates a } k : n-k-1 \text{ split,} \\ 0 & \text{otherwise.} \end{cases}$

 $E[X_k] = \Pr{\{X_k = 1\}} = 1/n$, since all splits are equally likely, assuming elements are distinct.

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Analysis (continued)

 $T(n) = \begin{cases} T(0) + T(n-1) + \Theta(n) & \text{if } 0 : n-1 \text{ split}, \\ T(1) + T(n-2) + \Theta(n) & \text{if } 1 : n-2 \text{ split}, \\ M \\ T(n-1) + T(0) + \Theta(n) & \text{if } n-1 : 0 \text{ split}, \end{cases}$

 $= \sum_{k=0}^{n-1} X_k \left(T(k) + T(n-k-1) + \Theta(n) \right)$ UNIVERSITY

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Calculating expectation

 $E[T(n)] = E' \sum_{\leq k=0}^{n-1} X_k (T(k) + T(n-k-1) + \Theta(n))_{\infty} / f$

Take expectations of both sides.

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Calculating expectation

 $E[T(n)] = E \sum_{\substack{i=0 \\ j \leq k=0}}^{n-1} X_k (T(k) + T(n-k-1) + \Theta(n))_{\infty}$ = $\sum_{k=0}^{n-1} E[X_k (T(k) + T(n-k-1) + \Theta(n))]$

Linearity of expectation.

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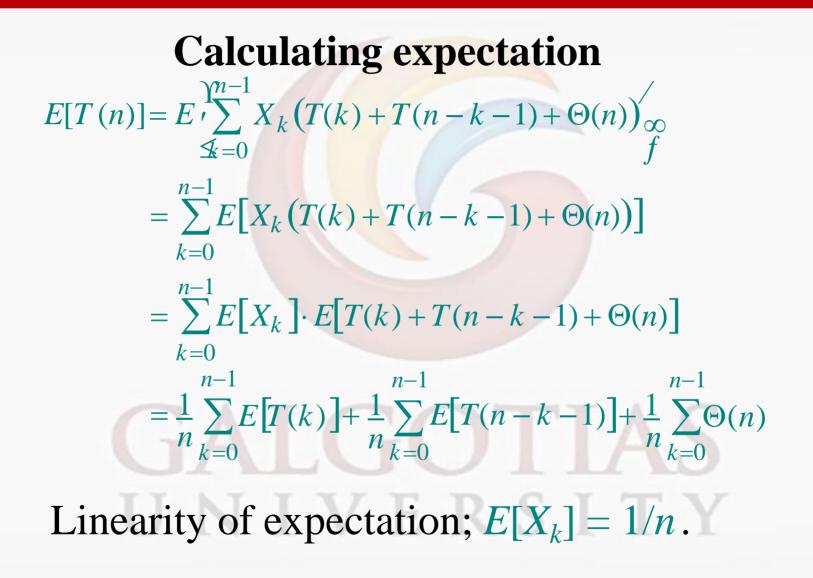
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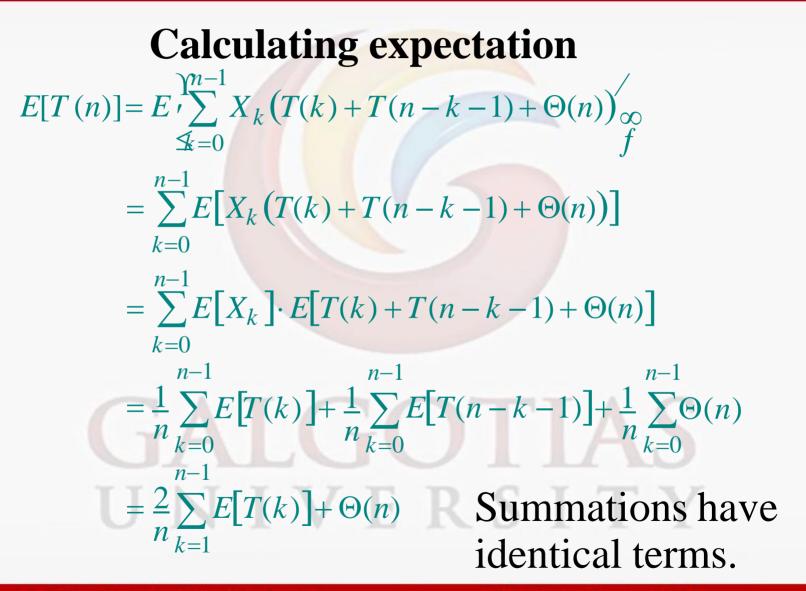
$\begin{aligned} & \textbf{Calculating expectation} \\ E[T(n)] = E \overset{\gamma n^{-1}}{\underset{k=0}{\overset{\sim}{\sum}} X_k \big(T(k) + T(n-k-1) + \Theta(n) \big)_{\infty} \\ & = \sum_{k=0}^{n-1} E \big[X_k \big(T(k) + T(n-k-1) + \Theta(n) \big) \big] \\ & = \sum_{k=0}^{n-1} E \big[X_k \big] \cdot E \big[T(k) + T(n-k-1) + \Theta(n) \big] \end{aligned}$

Independence of X_k from other random choices.

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Hairy recurrence $E[T(n)] = 2\sum_{k=2}^{\infty} E[T(k)] + \Theta(n)$

(The k = 0, 1 terms can be absorbed in the $\Theta(n)$.)

Prove: $E[T(n)] \le an \lg n$ for constant a > 0.

• Choose *a* large enough so that $an \lg n$ dominates E[T(n)] for sufficiently small $n \ge 2$.

Use fact: $\sum_{k=2}^{n-1} k \lg k \le \frac{1}{2} n^2 \lg n - \frac{1}{8} n^2$ (exercise).

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Substitution method

 $E[T(n)] \le \frac{2}{n} \sum_{k=2}^{n-1} ak \lg k + \Theta(n)$

Substitute inductive hypothesis.

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Substitution method $E[T(n)] \leq \frac{2}{n} \sum_{k=2}^{n-1} ak \lg k + \Theta(n)$ $\leq \frac{2a}{n} \ln^2 \lg n - \frac{1}{n} n^2 + \Theta(n)$ $n \ln 2 \qquad 8 \qquad \square$

Use fact.

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Substitution method n-1 $E[T(n)] \leq \frac{2}{\sum} ak \lg k + \Theta(n)$ $n_{k=2}$ $\leq \frac{2a}{4} \ln^2 \lg n - \frac{1}{2} \ln^2 \Theta n$ n 7 8 $= an \lg n - \Theta n$ Express as *desired – residual*. UNIVERSIT

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Substitution method n-1 $E[T(n)] \leq \frac{2}{\sum} ak \lg k + \Theta(n)$ $n_{k=2}$ $= \frac{2a}{n} \frac{1}{2} \ln^2 \lg n - \frac{1}{2} \ln^2 \frac{1}{2} \Theta n$ $= an \lg n - \frac{an}{4}$ $\leq an \lg n$. if *a* is chosen large enough so that an/4 dominates the $\Theta(n)$.

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Quicksort in Practice

- Quicksort is a great general-purpose sorting algorithm.
- Quicksort is typically over twice as fast as merge sort.
- Quicksort can benefit substantially from *code tuning*.
- Quicksort behaves well even with caching and virtual memory.

