School of Mechanical Engineering

Course Code : BTME3061

Course Name: FINITE ELEMENT ANALYSIS

Finite Element Analysis

BARS AND TRUSSES

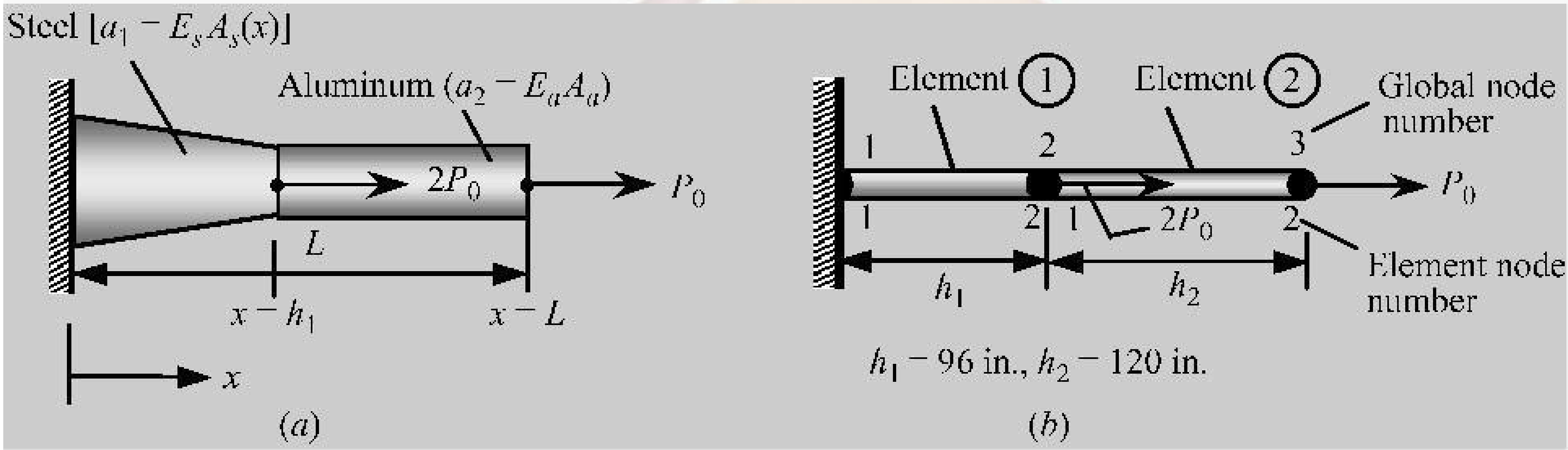
GALGOTIAS UNIVERSITY

Name of the Faculty: Mr. MANOJ KUMAR SHUKLA

Program Name: B.Tech (ME)

Lecture Objective-

- Analysis of structural problems of bar and truss using FEA methodology.



- Consider the bar shown in the above figure.
- It is composed of two different parts. One steel tapered part, and uniform Aluminum part.
- Calculate the displacement field using finite element method.

Bar Example

- The bar may be represented by two elements.
- The stiffness matrices of the two elements may be obtained using the following integration:

$$K_e = \int_{x_1}^{x_2} EA(x) \begin{Bmatrix} \frac{d\cdot_1}{dx} \\ \frac{d\cdot_2}{dx} \end{Bmatrix} \begin{bmatrix} \frac{d\cdot_1}{dx} & \frac{d\cdot_2}{dx} \end{bmatrix} dx = \int_{x_1}^{x_2} EA(x) \begin{bmatrix} \frac{1}{h_e^2} & \frac{-1}{h_e^2} \\ \frac{-1}{h_e^2} & \frac{1}{h_e^2} \end{bmatrix} dx$$

- For the Aluminum bar: E=10^7 psi, and A=1 in^2. we get:

$$K_{Al} = \frac{10^7}{120^2}\int_{x_1}^{x_2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} dx = \frac{10^7}{120}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

- For the Steel bar: E=3810^7 psi, and A=(1.5-0.5x/96) in^2. we get:

$$K_{Fe} = \frac{3.10^7}{96^2}\int_{x_1}^{x_2}\left(1.5 - \frac{0.5x}{96}\right)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} dx = \frac{4.75.10^7}{96}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

- Assembling the Stiffness matrix and utilizing the external forces, we get:

$$10^4 \begin{bmatrix} 57.8 & -8.33 \\ -8.33 & 8.33 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 2.10^5 \\ 10^5 \end{Bmatrix}$$

- The boundary conditions may be applied and the system of equations solved.

- Solving, we get:

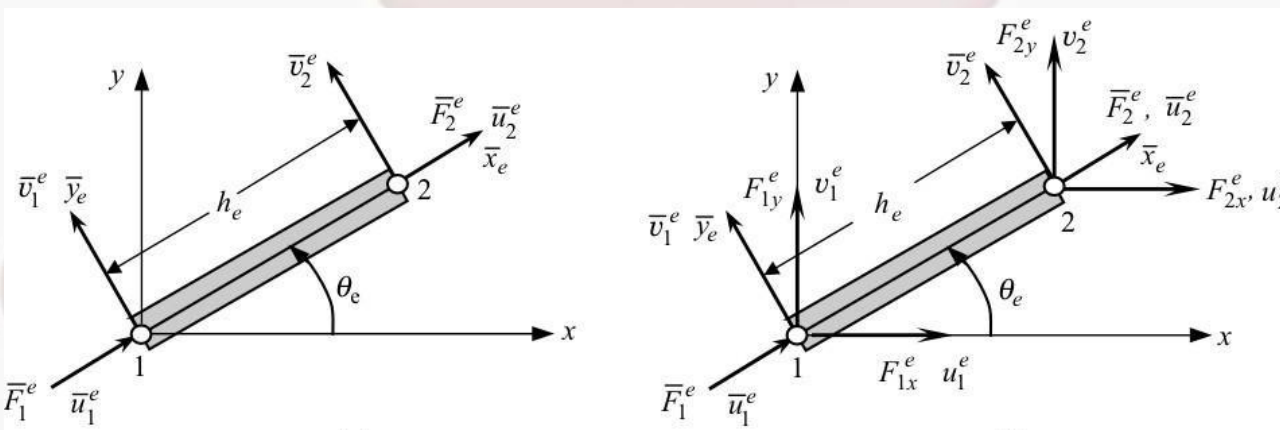
$$\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0.061 \\ 0.181 \end{Bmatrix} in$$

- For the secondary variables:

$$R = -30000 lb$$

Trusses

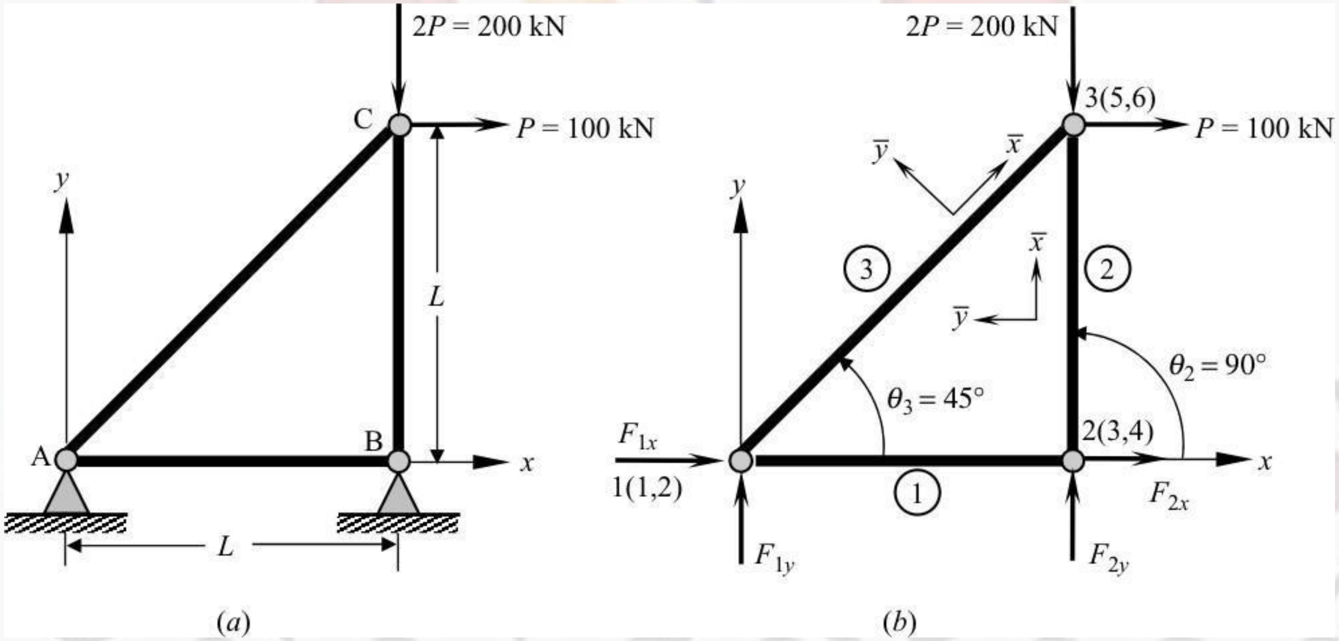
- A truss is a set of bars that are connected at frictionless joints.
- The Truss bars are generally oriented in the plain.



(a) (b)

- Now, the problem lies in the transformation of the local displacements of the bar, which are always in the direction of the bar, to the global degrees of freedom that are generally oriented in the plain.

- Use the finite element analysis to find the displacements of node C.



$$[K^1] = \frac{EA}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$[K^2] = \frac{EA}{L}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

$$[K^3] = \frac{EA}{L}\begin{bmatrix} 0.3536 & 0.3536 & -0.3536 & -0.3536 \\ 0.3536 & 0.3536 & -0.3536 & -0.3536 \\ -0.3536 & -0.3536 & 0.3536 & 0.3536 \\ -0.3536 & -0.3536 & 0.3536 & 0.3536 \end{bmatrix}$$

Assembly Procedure

$$[K]=\frac{EA}{L}\begin{bmatrix} 1.3536 & 0.3536 & -1 & 0 & -0.3536 & -0.3536 \\ 0.3536 & 0.3536 & 0 & 0 & -0.3536 & -0.3536 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ -0.3536 & -0.3536 & 0 & 0 & 0.3536 & 0.3536 \\ -0.3536 & -0.3536 & 0 & -1 & 0.3536 & 1.3536 \end{bmatrix}$$

Global Force Vector

$$\{F\} = \begin{Bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \end{Bmatrix} = \begin{Bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ P \\ -2P \end{Bmatrix}$$

Remember!

NO distributed load is applied to a truss

Boundary Conditions

$$U_1 = V_1 = U_2 = V_2 = 0$$

Remove the corresponding rows and columns

$$\frac{EA}{L}\begin{bmatrix} 0.3536 & 0.3536 \\ 0.3536 & 1.3536 \end{bmatrix}\begin{Bmatrix} U_3 \\ V_3 \end{Bmatrix} = \begin{Bmatrix} P \\ -2P \end{Bmatrix}$$

Continue! (as before)

Results

$$U_3 = 5.828 \frac{PL}{EA}, \quad V_3 = -\frac{3PL}{EA}$$

$$F_{1x} = -P, \quad F_{1y} = -P,$$

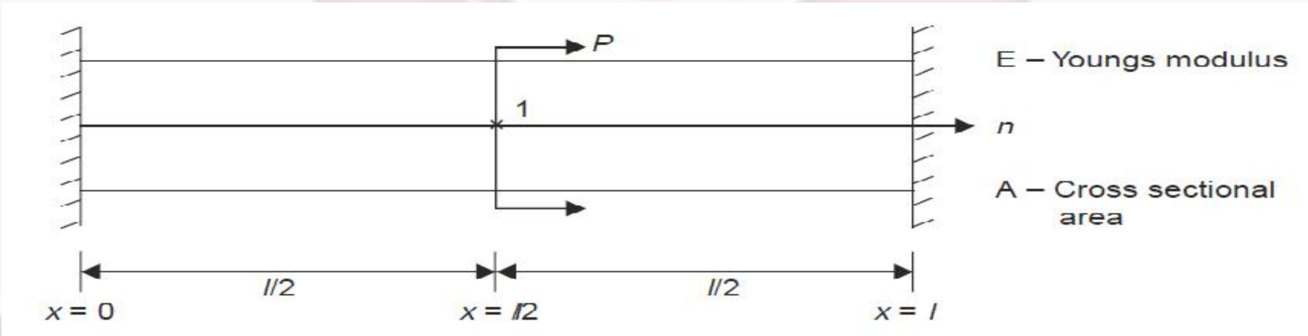
$$F_{2x} = 0, \quad F_{2y} = 3P$$

Summary

- In this lecture we learned how to apply the finite element modeling technique to bar problems with general orientation in a plain.

Questions-

Using Rayleigh-Ritz method, determine the expressions fordisplacement and stress in a fixed bar subject to axial force *P as shows in Fig. Draw the displacement and stress variation diagram. Take 3* terms in displacement function



Text Book-

1. Finite Element Analysis by S.S bhavikatti six multicolour edition,2018.New age International publisher. ISBN: 678-26-74589-23-4.

2. A Textbook of Finite Element Analysis Formulation and Programming by D.K.mahraj,Edition 2019. Publisher Willey India ISBN : 978-93-88425-93-3.

Reference Book-

1. Finite element analysis ,Theory and application with Ansys by Moaveni ,2nd edition 2015 ,publisher Pearson, ISBN- 528-43-88435-9.

2. Finite element Analysis By David V. Hutton ,Publisher Elizabeth A. Jomes ,4th edition 2017. ISBN: 0-07-23-9536-2

THANK YOU ;)