

## Duality Theory

- Every LP problem (called the 'Primal') has associated with another problem called the 'Dual'.
- The 'Dual' problem is an LP defined directly and systematically from the original (or Primal) LP model.
- The optimal solution of one problem yields the optimal solution to the other.
- Duality ease the calculations for the problems, whose number of variables is large.

## Rules for converting Primal to Dual

- If the Primal is to maximize, the dual is to minimize.
- If the Primal is to minimize, the dual is to maximize.
- For every constraint in the primal, there is a dual variable.
- For every variable in the primal, there is a constraint in the dual.

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## Dual Problem

### Primal LP :

$$\text{Max } z = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

subject to:

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \leq b_2$$

:

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \leq b_m$$

$$x_1 \geq 0, x_2 \geq 0, \dots, x_j \geq 0, \dots, x_n \geq 0.$$

### Associated Dual LP :

$$\text{Min. } z = b_1y_1 + b_2y_2 + \dots + b_my_m$$

subject to:

$$a_{11}y_1 + a_{21}y_2 + \dots + a_{m1}y_m \geq c_1$$

$$a_{12}y_1 + a_{22}y_2 + \dots + a_{m2}y_m \geq c_2$$

:

$$a_{1n}y_1 + a_{2n}y_2 + \dots + a_{mn}y_m \geq c_n$$

$$y_1 \geq 0, y_2 \geq 0, \dots, y_j \geq 0, \dots, y_m \geq 0.$$

## Example

### Primal

$$\text{Min.. } Z = 10x_1 + 15x_2$$

Subject to constraints:

$$5x_1 + 7x_2 \geq 80$$

$$6x_1 + 11x_2 \geq 100$$

$$x_1, x_2 \geq 0$$

### Dual

$$\text{Max.. } Z' = 80y_1 + 100y_2$$

Subject to constraints:  $5y_1 + 6y_2 < 10$

$$7y_1 + 11y_2 < 15 \quad y_1, y_2 \geq 0$$

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# School of basic and applied Sciences

Course Code : MSCM302

Course Name: Operation Research

Youtube/other Video Links

- <https://youtu.be/M8POtpPtQZc>
- <https://youtu.be/8IRrgDoV8Eo>
- <https://youtu.be/YrsbJG8XqU0>

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Program Name: M.Sc.

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