

### Lecture Notes

on

### Information Theory and Coding

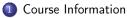


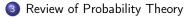
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Information Measures



What is Information Theory?

- IT is a branch of math (a strictly deductive system). (C. Shannon, The bandwagon)
- $\bullet$  General statistical concept of communication. (N. Wiener, What is IT?)
- It was build upon the work of Shannon (1948)
- It answers to two fundamental questions in Communications Theory:
  - What is the fundamental limit for information compression?
  - What is the fundamental limit on information transmission rate over a communications channel?



What is Information Theory?

- Mathematics: Inequalities
- Computer Science: Kolmogorov Complexity
- Statistics: Hypothesis Testings
- Probability Theory: Limit Theorems
- Engineering: Communications
- Physics: Thermodynamics
- Economics: Portfolio Theory



**Communications Systems** 

• The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point. (Claude Shannon: A Mathematical Theory of Communications, 1948)



Digital Communications Systems

- Source
- Source Coder: Convert an analog or digital source into bits.
- Channel Coder: Protection against errors/erasures in the channel.
- Modulator: Each binary sequence is assigned to a waveform
- Channel: Physical Medium to send information from transmitter to receiver. Source of randomness.
- Demodulator, Channel Decoder, Source Decoder, Sink.



Digital Communications Systems

- Modulator + Channel = Discrete Channel.
- Binary Symmetric Channel.
- Binary Erasure Channel.



Review of Probability Theory

- Axiomatic Approach
- Relative Frequency Approach



# Axiomatic Approach

- Application of a mathematical theory called *M*easure Theory.
- It is based on a triplet

$$(\Omega, \mathcal{F}, P)$$

where

- $\Omega$  is the sample space, which is the set of all possible outcomes.
- *F* is the *σ*-algebra, which is the set of all possible events (or combinations of outcomes).
- *P* is the probability function, which can be any set function, whose domain is Ω and the range is the closed unit interval [0,1]. It must obey the following rules:
  - $P(\Omega) = 1$
  - Let A be any event in  $\mathcal{F}$ , then  $P(A) \ge 0$ .
  - Let A and B be two events in  $\mathcal{F}$  such that  $A \cap B = \emptyset$ , then  $P(A \cup B) = P(A) + P(B)$ .



Axiomatic Approach: Other properties

- Probability of complement:  $P(\overline{A}) = 1 P(A)$ .
- $P(A) \le 1$ .
- $P(\emptyset) = 0.$
- $P(A \cup B) = P(A) + P(B) P(A \cap B).$



## Conditional Probability

• Let A and B be two events, with P(A) > 0. The conditional probability of B given A is defined as:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

- Hence,  $P(A \cap B) = P(B|A)P(A) = P(A|B)P(B)$
- If  $A \cap B = \emptyset$  then P(B|A) = 0.
- If  $A \subset B$ , then P(B|A) = 1.