Lecture Notes

on

Information Theory and Coding

GALGOTIAS UNIVERSITY
(Established under Galgotias University Uttar Pradesh Act No. 14 of 2011)

July 2020
(Be safe and stay at home)

What is Information?

- It is a measure that quantifies the *uncertainty* of an event with given probability - Shannon 1948.
- For a discrete source with finite alphabet $\mathcal{X} = \{x_0, x_1, \ldots, x_{M-1}\}$ where the probability of each symbol is given by $P(X = x_k) = p_k$

$$I(x_k) = \log \frac{1}{p_k} = -\log(p_k)$$

- If logarithm is base 2, information is given in bits.

What is Information?

- It represents the *surprise* of seeing the outcome (a highly probable outcome is not surprising).

event	probability	surprise
one equals one	1	0 bits
wrong guess on a 4-choice question	3/4	0.415 bits
correct guess on true-false question	1/2	1 bit
correct guess on a 4-choice question	1/4	2 bits
seven on a pair of dice	6/36	2.58 bits
win any prize at Euromilhões	1/24	4.585 bits
win Euromilhões Jackpot	$\approx 1/76$ million	≈ 26 bits
gamma ray burst mass extinction today	$< 2.7 \cdot 10^{-12}$	> 38 bits

Entropy

- Expected value of information from a source.

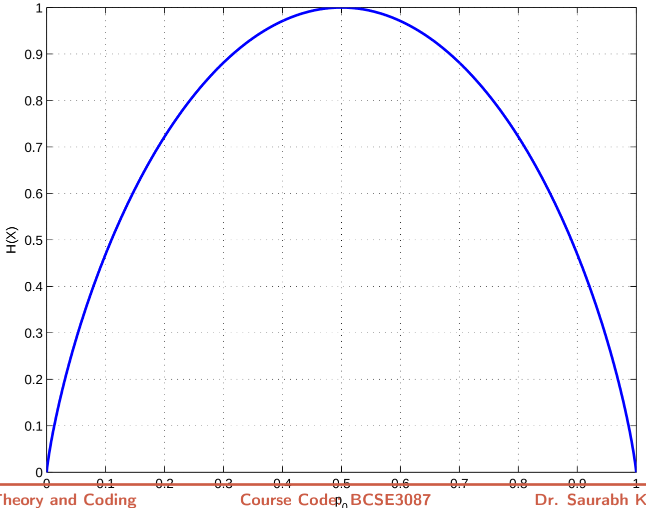
$$H(X) = E[I(x_k)] = \sum_{x \in \mathcal{X}} p_x(x) I(x_k)$$
$$= -\sum_{x \in \mathcal{X}} p_x(x) \log p_x(x)$$

Entropy of binary source

- Let X be a binary source with p_0 and p_1 being the probabililty of symbols x_0 and x_1 respectively.

$$
\begin{aligned}
H(X) &= -p_0 \log p_0 - p_1 \log p_1 \\
&= -p_0 \log p_0 - (1 - p_0) \log(1 - p_0)
\end{aligned}
$$

Entropy of binary source



Joint Entropy

- The joint entropy of a pair of random variables X and Y is given by:.

$$H(X,Y) = -\sum_{y\in\mathcal{Y}}\sum_{x\in\mathcal{X}} p_{XY}(x,y)\log p_{X,Y}(x)$$