Lecture Notes

on

Master's Method

July 2020

(Be safe and stay at home)

Master's Method

To apply master's method, recurrence relation should be in the form of:
$T(n) = a.T(\frac{n}{b}) + \theta(n^k log^p n)$; where a>=1, b>1, k>=0 & p-> real number

1. if a>b$^k then$T(n)=$\theta(n^{log_b^a})$
2. if a<b$^k then$

2.1 if p>=0 then $T(n) = \theta(n^k log^p n)$
2.2 if p<0 then $T(n) = O(n^k)$

3. if a=b$^k then$

3.1 if p>-1 then $T(n) = \theta(n^{log_b^a} log^{p+1} n)$
3.2 if p=-1 then $T(n) = \theta(n^{log_b^a} loglogn)$
3.3 if p<-1 then $T(n) = \theta(n^{log_b^a})$

Problem 01:

$T(n) = 3T(\frac{n}{2}) + n^2$

compare it with: $T(n) = a.T(\frac{n}{b}) + \theta(n^k log^p n)$

a=3, b=2, k=2, p=0
$a < b^k$ ...$3 < 4$ and $p >= 0$
Apply case 2.1
$T(n) = \theta(n^k log^p n)$
$= \theta(n^2 log^0 n)$
$= \theta(n^2)$

Problem 02:

$T(n) = 4T(\frac{n}{2}) + n^2$

compare it with: $T(n) = a.T(\frac{n}{b}) + \theta(n^k log^p n)$

a=4, b=2, k=2, k=2, p=0
$a = b^k$ ...$4 = 2^2$ and $p > -1$
Apply case 3.1
$T(n) = \theta(n^{log_b^a} log^{p+1} n)$
$= \theta(n^{log_2^{2^2}} log^{0+1} n)$
$= \theta(n^2 logn)$

Problem 03:

$T(n) = T(\frac{n}{2}) + n^2$

compare it with: $T(n) = a.T(\frac{n}{b}) + \theta(n^k log^p n)$

a=1, b=2, k=2, p=0
$a < b^k$ ...$1 = 2^2$ and $p >= 0$
Apply case 2.1
$T(n) = \theta(n^k log^p n)$
$= \theta(n^2 log^0 n) = \theta(n^2)$

Problem 04:

$T(n) = 16T(\frac{n}{4}) + n$

compare it with: $T(n) = a.T(\frac{n}{b}) + \theta(n^k log^p n)$

a=16, b=4, k=1, p=0
$a > b^k$ ...$16 = 4^1$
Apply case 1
$T(n) = \theta(n^{log_b^a})$
$= \theta(n^{log_4^{4^2}})$
$= \theta(n^2)$

Problem 05:

$T(n) = 2T(\frac{n}{2}) + nlogn$

compare it with: $T(n) = a.T(\frac{n}{b}) + \theta(n^k log^p n)$

a=2, b=2, k=1, p=1
$a = b^k$ ...$2 = 2^1$ & p>-1
Apply case 3.1
$T(n) = \theta(n^{log_b^a} log^{p+1} n)$
$= \theta(n^{log_2^2} log^{1+1} n)$
$= \theta(nlog^2 n)$

Cases where Master's method doesn't apply

1. $T(n) = 2^n T(\frac{n}{2}) + n^n$
   a should be constant.
2. $T(n) = 0.5T(\frac{n}{2}) + \frac{1}{n}$
   a should be greater than or equal to 1.
3. $T(n) = 64T(\frac{n}{8}) - n^2 logn$
   function should be positive
4. $T(n) = 2^n T(\frac{n}{2}) + n^n$
   a is not constant.

Practice Questions

1. $T(n) = T(\frac{n}{2}) + c$
2. $T(n) = 3T(\frac{n}{4}) + nlogn$
3. $T(n) = 2T(\frac{n}{2}) + \frac{n}{logn}$
4. $T(n) = 2T(\frac{n}{4}) + n^{0.51}$
5. $T(n) = 6T(\frac{n}{3}) + n^2 logn$
6. $T(n) = 7T(\frac{n}{3}) + n^2$
7. $T(n) = 3T(\frac{n}{3}) + \sqrt{n}$

Q & A?

Queries are welcome on slack channel for discussion