# Lecture Notes

on

# Counting & Radix Sort

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**(Be safe and stay at home)**

# Counting Sort

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2. The counting information stored in $C$ can be used to determine the position of each element in the sorted array. Suppose we modify the values of the $C[j]$ so that *now*

   $C[j] =$ the number of keys *less than or equal* to $j$.

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   Then we know that the elements with key "$j$" must be stored at the indices $C[j-1]+1, \ldots, C[j]$ of the final sorted array.
3. We use a "trick" to move the elements to the right position of an auxiliary array. Then we copy the sorted auxiliary array back to the original one.

# Implementation of Counting Sort

**Algorithm** COUNTING SORT($A, m$)

```
n ← A.length
Initialise array C[1 ... m]
for i ← 1 to n do
        j ← A[i].key
        C[j] ← C[j] + 1
for j ← 2 to m do
        C[j] ← C[j] + C[j − 1]     ▷ C[j] stores ♯ of keys ≤ j
Initialise array B[1 ... n]
for i ← n downto 1 do
        j ← A[i].key            ▷ A[i] highest w. key j
        B[C[j]] ← A[i]          ▷ Insert A[i] into highest free index for j
        C[j] ← C[j] − 1
for i ← 1 to n do
        A[i] ← B[i]
```

# Analysis of Counting Sort

- The loops in lines 3–5, 9–12, and 13–14 all require time $\Theta(n)$.
- The loop in lines 6–7 requires time $\Theta(m)$.
- Thus the overall running time is

$$O(n + m).$$

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Note: COUNTING-SORT is STABLE.

- (*After* sorting, 2 items with the same key have their *initial relative order*).

# Radix Sort

## Basic Assumption

Keys are sequences of digits in a fixed range $0, \ldots, R-1$, all of equal length $d$.

**Examples** of such keys

- 4 digit hexadecimal numbers (corresponding to 16 bit integers)
  $R = 16, d = 4$
- 5 digit decimal numbers (for example, US post codes)
  $R = 10, d = 5$
- Fixed length ASCII character sequences
  $R = 128$
- Fixed length byte sequences
  $R = 256$

# Stable Sorting Algorithms

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**Examples**

- COUNTING-SORT, MERGE-SORT, and INSERTION SORT are all stable.
- QUICKSORT is not stable.
- If keys and elements are exactly the same thing (in our setting, an element is a structure containing the key as a sub-element) then we have a much easier (non-stable) version of COUNTING-SORT. (How? ... CLASS?).

# Radix Sort (cont'd)

**Idea**

Sort the keys digit by digit, *starting with the least significant digit*.

**Example**

| | | | | | | |
|---|---|---|---|---|---|---|
| now | | sob | | tag | | ace |
| for | | nob | | ace | | bet |
| tip | | ace | | bet | | dim |
| ilk | | tag | | dim | | for |
| dim | | ilk | | tip | | hut |
| tag | | dim | | sky | | ilk |
| jot | ⟶ | tip | ⟶ | ilk | ⟶ | jot |
| sob | | for | | sob | | nob |
| nob | | jot | | nob | | now |
| sky | | hut | | for | | sky |
| hut | | bet | | jot | | sob |
| ace | | now | | now | | tag |
| bet | | sky | | hut | | tip |

Each of the three sorts is carried out with respect to the digits in that column. "Stability" (and having previously sorted digits/suffixes to the right), means this achieves a sorting of the suffixes starting at the current column.

# Radix Sort (cont'd)

**Algorithm** RADIX-SORT($A, d$)

1. **for** $i \leftarrow 0$ **to** $d$ **do**
2. use stable sort to sort array $A$ using digit $i$ as key

Most commonly, COUNTING SORT is used in line 2 - this means that once a set of digits is already in sorted order, then (by stability) performing COUNTING SORT on the *next-most significant* digits preserves that order, within the "blocks" constructed by the new iteration.

Then each execution of line 2 requires time $\Theta(n + R)$.
Thus the overall time required by RADIX-SORT is

$$\Theta(d(n + R))$$

# Sorting Integers with Radix-Sort

**Theorem 2**

*An array of length n whose keys are b-bit numbers can be sorted in time*

$$\Theta(n\lceil b/\lg n\rceil)$$

*using a suitable version of* RADIX-SORT.

**Proof:** Let the digits be blocks of $\lceil \lg n\rceil$ bits. Then $R = 2^{\lceil \lg n\rceil} = \Theta(n)$ and $d = \lceil b/\lceil \lg n\rceil\rceil$. Using the implementation of RADIX-SORT based on COUNTING SORT the integers can be sorted in time

$$\Theta(d(n+R)) = \Theta(n\lceil b/\lg n\rceil).$$

Note: If all numbers are at most $n^k$, then $b = k \lg n \ldots \Rightarrow$ Radix Sort is $\Theta(n)$ (assuming $k$ is some constant, eg 3, 10).